10/ May 2000 Course 2 Exam

$$aeir = .125$$
$$v = \frac{1}{1.125}$$

$$I_n = 153.86 = R(1-v) \Longrightarrow R = 1384.74$$

$$\sum_{K=1}^{n-1} P_K = 6009.12$$
$$= Rv^n + Rv^{n-1} + \cdots + Rv^2$$
$$= Rv\left(v + v^2 + \cdots + v^{n-1}\right)$$
$$= \frac{1384.74}{1.125} \cdot a_{\overline{n-1}|.125} = 6009.12$$

$$\Longrightarrow n-1 = 8 \Longrightarrow n = 9$$

$$\therefore L = 1384.74\, a_{\overline{9}|.125} \doteq 7240.09$$

$$\Longrightarrow I_1 = .125(L) \doteq 905.01$$

$$\Longrightarrow P_1 = Y = 1384.74 - I_1 \doteq 479.73$$

24 / (May 2000 Course 2 Exam)

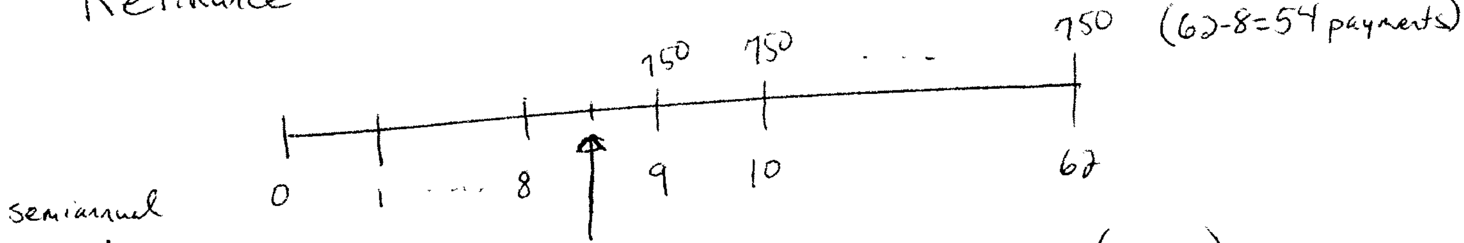
Original: $qeir = .03$

$\Longrightarrow seir = (1.03)^2 - 1 = .0609$

$$12000 = 750\, a_{\overline{n}|.0609} \Longrightarrow n \doteq 62$$

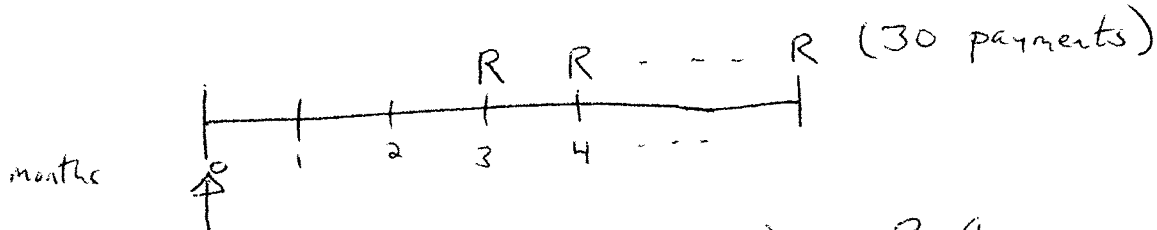
↳ close enough; just use 62

Refinance:



$$\text{Balance} = \underbrace{750\, a_{\overline{54}|.0609}}_{\text{time 8 value}} (1.03)$$

New timeline: ~~meir~~ $meir = \frac{.09}{12} = .0075$



$$\text{Balance} = 750\, a_{\overline{54}|.0609} (1.03) = R\, a_{\overline{30}|.0075} \cdot v^2_{.0075}$$

$$\Longrightarrow R \doteq 461.13$$

26 / May 2000 Course 2 Exam

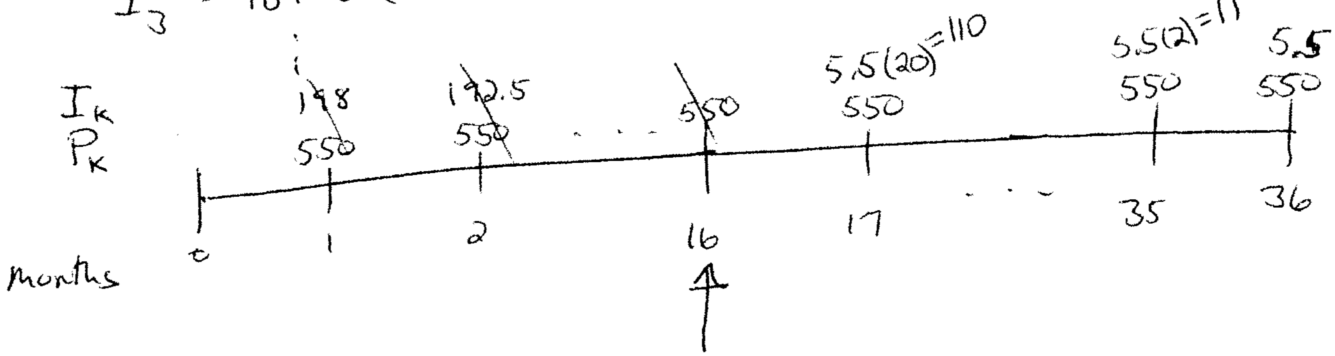
meir $= .01$

$\frac{19800}{36} = 550 = P_k \qquad k = 1, 2, \ldots, 36$

$I_1 = 19800(.01) = 198 \qquad B_1 = 19800 - 550 = 19250$

$I_2 = 19250(.01) = 192.5 \qquad B_2 = 19250 - 550 = 18700$

$I_3 = 18700(.01) = 187 \qquad \ldots$



P = Price to Bank Y

yield $= .07$ seir

$\therefore$ meir $= (1.07)^{1/6} - 1 = i$

$$P = 550\, a_{\overline{20}|i} + 5.5\,(Da)_{\overline{20}|i}$$

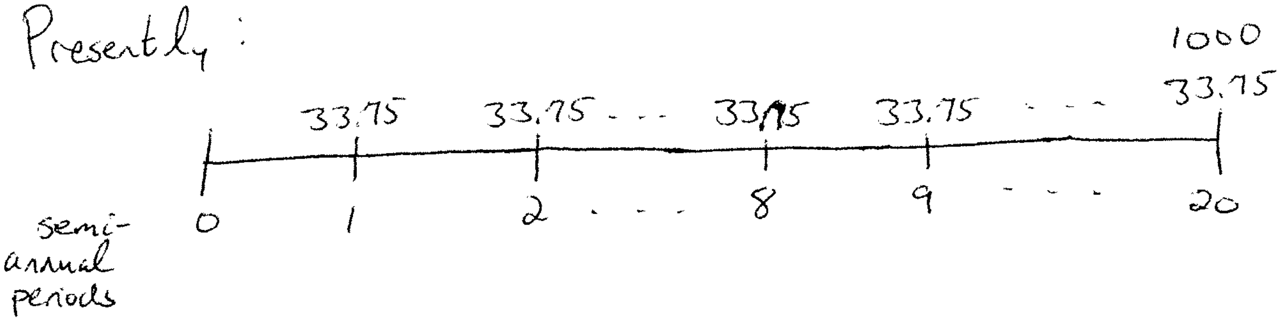
$$P \doteq 10857.28$$

29/ (May 2000 Course 2 Exam)

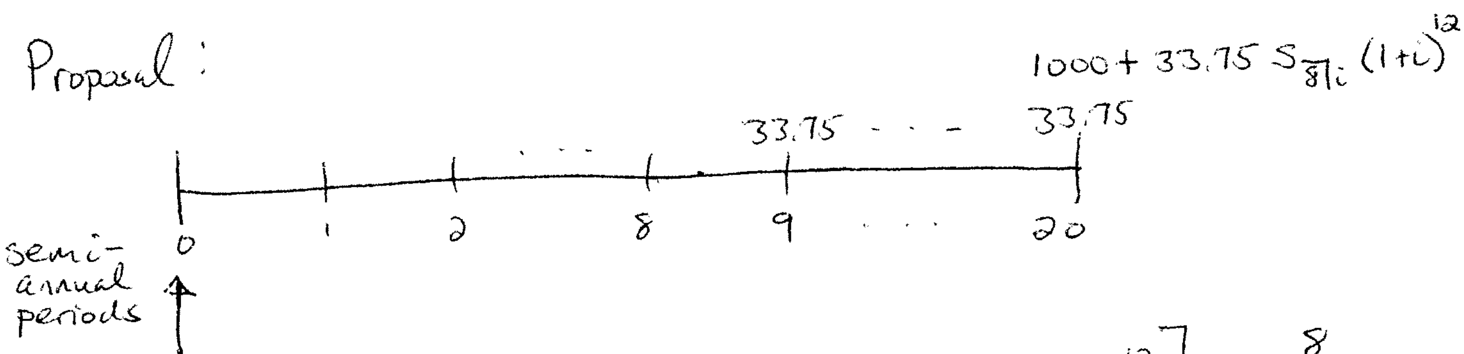
$r = \frac{.0675}{2} = .03375$

$i = \frac{.074}{2} = .037$

Presently:



Proposal:



$$P = \left[33.75\, a_{\overline{12}|i} + \left(1000 + 33.75\, s_{\overline{8}|i}(1+i)^{12}\right)\cdot v^{12}\right]\cdot v^{8}$$

$$= \underline{33.75\, a_{\overline{12}|i}\, v^{8}} + 1000\, v^{20} + \underline{33.75\, a_{\overline{8}|i}}$$

$$= \underline{33.75\, a_{\overline{20}|i}} + 1000\, v^{20} = 954.63$$

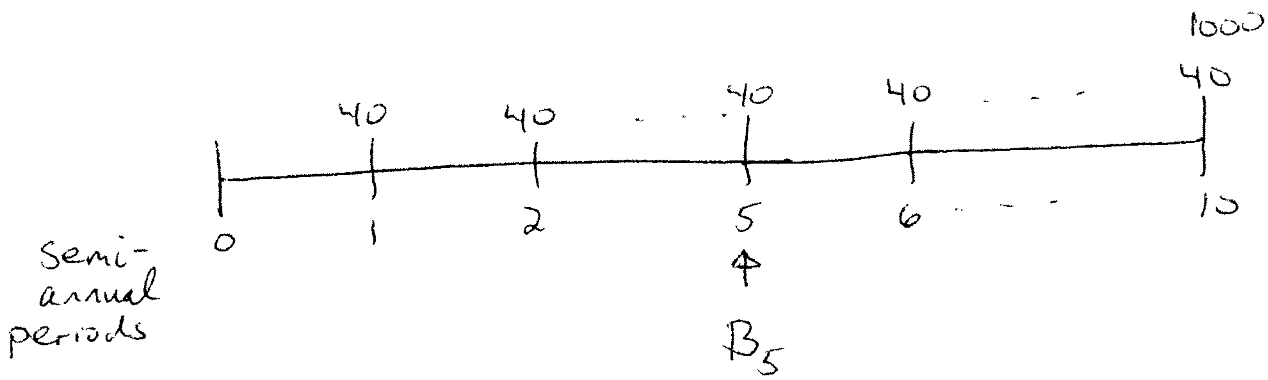
= same as the price before the proposal

(makes sense, since the missing coupons are replaced, with interest at the same rate as the yield on the bond)

43 / May 2000 Course 2 Exam

$r = .04$

$i = seir = .0375$



$$B_5 = 40\, a_{\overline{5}|,.0375} + 1000\, v^5 \doteq 1011.21$$

$$I_6 = i \cdot B_5 \doteq .0375(1011.21) \doteq 37.92$$

$$P_6 = Fr - I_6 \doteq 40 - 37.92 = 2.08$$

12 (Nov. 2000 Course 2 Exam)

$$L = 1000\, a_{\overline{10|}i}$$

$$\sum_{K=1}^{10} I_K = 10R - L$$

$$\therefore \quad L = 10(1000) - L$$

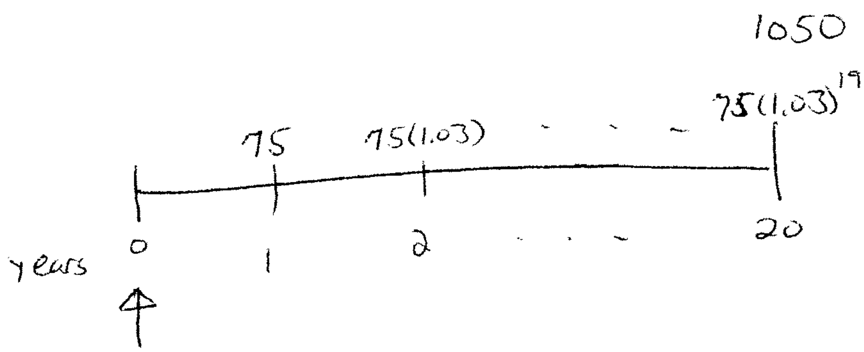
$$\Longrightarrow \quad L = 5000$$

$$\therefore \quad 5000 = 1000\, a_{\overline{10|}i}$$

$$\Longrightarrow i \doteq 15.1\%$$

$$I_1 = i \cdot L \doteq (.151)(5000) = 755$$

30 / Nov. 2000
Course 2 Exam



aeir = .0825

$$P \overset{\text{VEP}}{=} 75v + 75(1.03)v^2 + \cdots \text{ (20 terms)} + 1050v^{20}$$

$$= 1050\, v^{20}_{.0825} + \frac{75}{1.0825}\left(1 + \frac{1.03}{1.0825} + \cdots \text{ (20 terms)}\right)$$

$$= 1050\, v^{20}_{.0825} + \frac{75}{1.0825} \cdot \ddot{a}_{\overline{20}|\left(\frac{1.0825}{1.03}-1\right)}$$

$$\doteq 215.10 + 900.02 = 1115.12$$

34 / Nov. 2000 Course 2 Exam

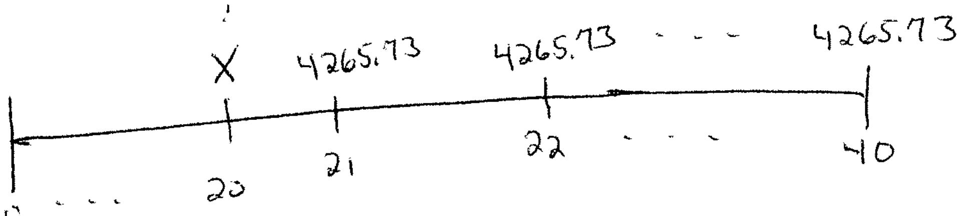
$$150000 = 5483.36\, a_{\overline{40}|.02} \quad \checkmark \text{ (given)}$$

$$B_{12} = 116692.92 \quad \text{(I changed N to 12 and computed FV (i.e. Retro))}$$

$$116692.92 = 5134.62\, a_{\overline{28}|.015} \quad \checkmark \text{ (given)}$$

$$B_{20} \overset{\text{Pro}}{=} 5134.62\, a_{\overline{20}|.015} = 88154.44$$

$$j_{eir} = \frac{.07}{4} = .0175$$



$$B_{20} = 88154.44$$

$$\therefore\ 88154.44 = X + 4265.73\, a_{\overline{20}|.0175}$$

$$\Longrightarrow X = 16691.17$$

40 / Nov. 2000 Course 2 Exam

$P_2 = 977.19$ $\qquad$ $P_4 = 1046.79$

$P_2(1+i)^2 = P_4 \implies i \doteq .035 =$ seir (yield rate)

$$\text{Premium} = \sum_{K=1}^{30} P_K = P_1 \cdot s_{\overline{30|}.035}$$

$$= P_1 + P_1(1+i) + \cdots + P_1(1+i)^{29} = P_1[1 + (1+i) + \cdots + (1+i)^{29}]$$

$$P_1 = \frac{P_2}{1+i} = \frac{977.19}{1.035}$$

$$\therefore \text{Premium} = \frac{977.19}{1.035} \cdot s_{\overline{30|}.035} \doteq 48739.29$$

48/ Nov 2000 Course 2 Exam

$$B_{12} = 8000(1.08)^{12} - 800\, s_{\overline{12}|,08} \doteq 4963.66$$

The SF deposits need to accumulate to B_{12}

$$\therefore\ 4963.66 = X\, s_{\overline{12}|,04}$$

$$\Rightarrow X \doteq 330.34$$

55/ (Nov. 2000 Course 2 Exam)

Using Lump Sum Method:

$$\text{Amount of Interest Paid} = X(1.06)^{10} - X$$
$$= X\left[(1.06)^{10} - 1\right] = A$$

Using Amortization Method: $X = R \cdot a_{\overline{10}|i}$ $\quad i = .06$

$$\text{Amount of Interest Paid} = 10R - X$$
$$= 10\left(\frac{X}{a_{\overline{10}|}}\right) - X = B$$

$$\therefore A - B = 356.54$$
$$\left[X(1.06)^{10} - \cancel{X}\right] - \left[\frac{10X}{a_{\overline{10}|}} - \cancel{X}\right] = 356.54$$
$$\therefore X\left[(1.06)^{10} - \frac{10}{a_{\overline{10}|.06}}\right] = 356.54$$
$$\Longrightarrow X \doteq 825$$

4 / (May 2001 Course 2 Exam)

i) $20000 = X\, a_{\overline{20}|.065}$

$\Rightarrow X = 1815.13$

ii) $R^{I} = 20000(.08) = 1600$

$\therefore R^{SF} = 1815.13 - 1600 = 215.13$

$215.13\, s_{\overline{20}|j} = 20000$

$\Rightarrow j \doteq 14.18\%$

7/ May 2001 Course 2 Exam

seir = .06

S: $5000(1.06)^{10} \doteq 8954.24$

Seth pays $8954.24 - 5000 = 3954.24$ in interest

J: Janice pays $5000(.06) = 300$ every 6 months for 5 years, totaling 3000 in interest

L: $5000 = R\, a_{\overline{10}|.06}$ (Amortization Method)

$\Rightarrow R \doteq 679.34$

Lori Pays $10R - L \doteq 6793.4 - 5000 = 1793.40$ in interest

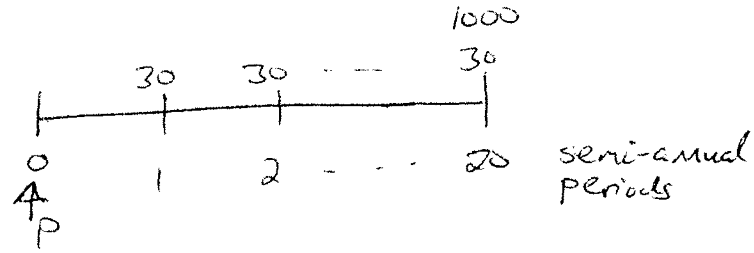
Total amount of interest on all three loans is

$3954.24 + 3000 + 1793.40 = 8747.64$

41 / May 2001 Course 2 Exam

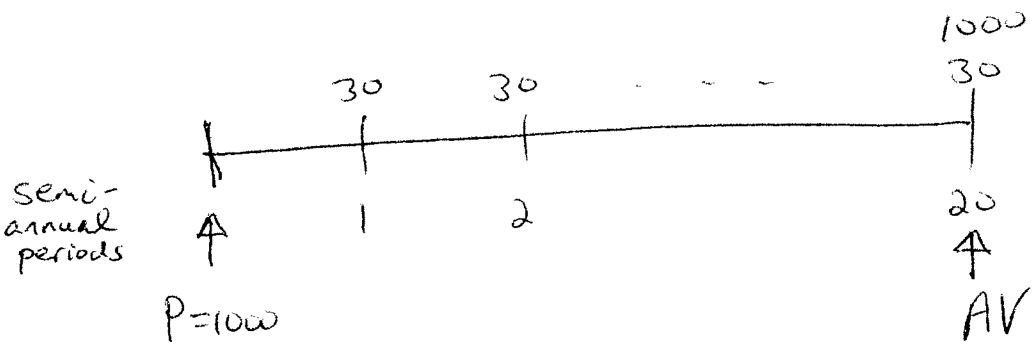
$r = .03$

$i = .03$



$$P = 30\,a_{\overline{20}|.03} + 1000\,v^{20}_{.03}$$

$\therefore\ P = 1000$ (price Bill paid)



Bill's yield is 7% aeir!

j = seir

$$AV = 30\,s_{\overline{20}|j} + 1000 = 1000(1.07)^{10}$$

$$\Longrightarrow\ j \doteq 4.76\%$$

i = aeir $\Longrightarrow\ i = (1+j)^2 - 1 \doteq 9.75\%$

6/Nov. 2001 Course 2 Exam) (i) $2000 = R\, a_{\overline{10}|.0807} \Rightarrow R \doteq 299.00$

Sum of payments under option (i) is 2990

(ii) $P_K = 200 \quad K = 1, 2, \ldots, 10$

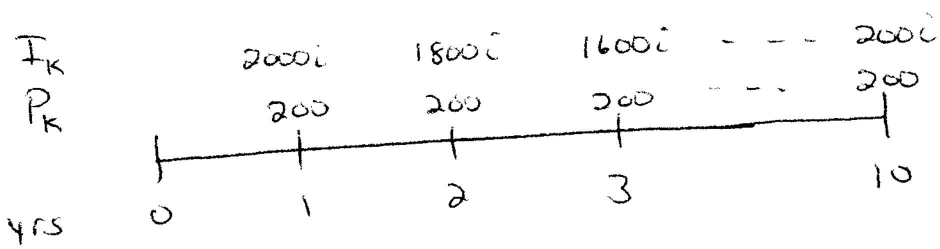
$I_1 = 2000 \cdot i \qquad B_1 = 2000 - 200 = 1800$

$I_2 = 1800\, i \qquad B_2 = 1800 - 200 = 1600$

$\ldots$

$I_3 = 1600\, i$

$\vdots$



Sum of payments under option (ii) is

$200(10) + (200i + 400i + \cdots + 1800i + 2000i)$

$= 2000 + 200i(1 + 2 + \cdots + 10)$

$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

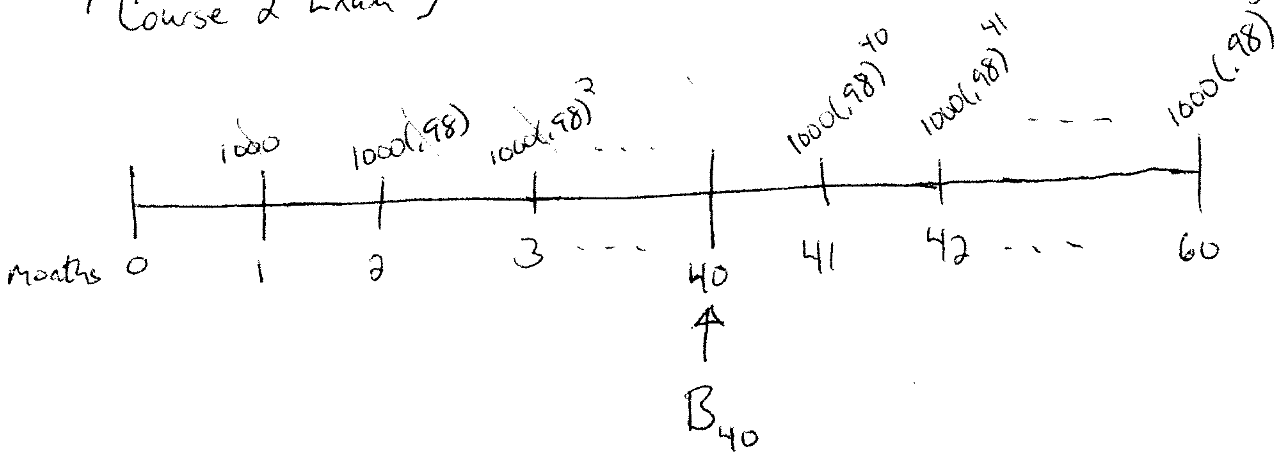
$= 2000 + 200i(55)$

$= 2000 + 11000\, i$

$\therefore\ 2000 + 11000\, i = 2990 \Rightarrow i = .09$

9 / Nov. 2001 Course 2 Exam

meir $= \frac{.09}{12} = .0075$



$$B_{40} \overset{\text{Pro}}{\underset{\text{VEP}}{=}} 1000(.98)^{40} v + 1000(.98)^{41} v^2 + \cdots \text{ (20 terms)}$$

$$= \frac{1000(.98)^{40}}{1.0075} \left[1 + \frac{.98}{1.0075} + \cdots \text{ (20 terms)} \right.$$

$$= \frac{1000(.98)^{40}}{1.0075} \cdot \ddot{a}_{\overline{20}|\left(\frac{1.0075}{.98} - 1\right)}$$

$$= 6889.11$$

31 / Nov. 2001 Course 2 Exam

i = seir

Bond X: $P = 1000r\, a_{\overline{2n}|} + \underbrace{C v_i^{2n}}_{= PV(\text{redemption value})}$ $\therefore C v_i^{2n} = 381.50$

$= 1000r\left(\frac{1-v^{2n}}{i}\right) + 381.50$

$= 1000\left(\frac{r}{i}\right)(1-v^{2n}) + 381.50$

$= 1000(1.03125)(1-v^{2n}) + 381.50$

$= 1031.25 - 1031.25 v^{2n} + 381.50$

$= 1412.75 - 1031.25 v_i^{2n}$

Bond Y: (This is a zero-coupon bond, and so the price is equal to the PV of the redemption value.)

$P = 647.80 = C v_j^{n/2}$ j = aeir

$= C v_i^n$ where i = seir above

$\therefore 647.80 = C v_i^n$

From information about Bond X, $C v_i^{2n} = 381.5$

$\therefore 381.5 = C v_i^n \cdot v_i^n = 647.8 v_i^n \Rightarrow v_i^n = \frac{381.5}{647.8}$

$\therefore$ for Bond X,

$P = 1412.75 - 1031.25\left(\frac{381.5}{647.8}\right)^2$

$\doteq 1055.09$

15/May 2003 Course 2 Exam

$$1000 = P\, a_{\overline{10}|.10} \implies P \doteq 162.75$$

Using the SF, $R^I = 1000(.1) = 100$

$$\implies R^{SF} \doteq 62.75$$

The SF balance at the end of year 10 is

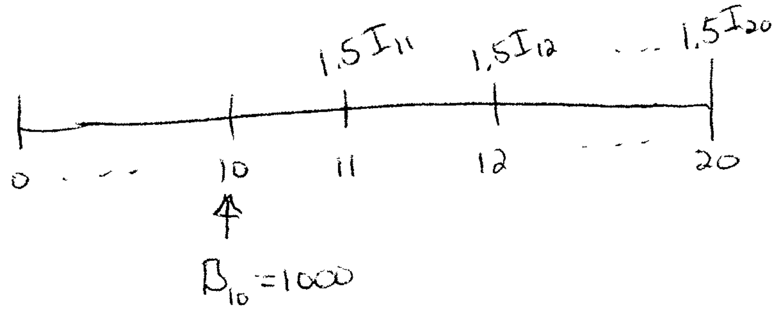
$$B_{10}^{SF} = R^{SF} \cdot s_{\overline{10}|.14} \doteq 62.75\, s_{\overline{10}|.14} \doteq 1213.42$$

After repaying the loan of 1000, the balance is 213.42.

$i = aeir = .10$

39/ May 2003 Course 2 Exam) Since each of the first 10 payments is interest only, then $B_{10} = B_9 = \cdots = B_1 = B_0 = 1000$

For the next 10 payments, we have



$I_{11} = i \cdot B_{10}$ $\quad R_{11} = 1.5 \cdot i \cdot B_{10} \overset{i=.1}{=} .15 B_{10}$ $\quad B_{11} = B_{10}(1+i) - R_{11}$

$= 1.1 B_{10} - .15 B_{10} = .95 B_{10}$

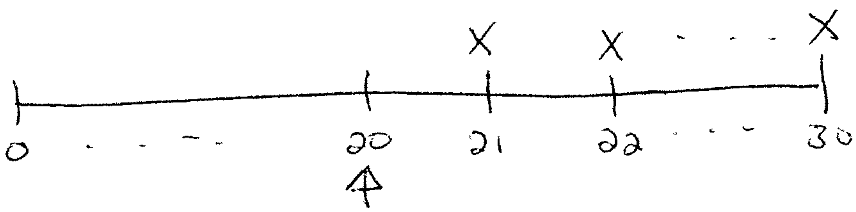
$I_{12} = .1 B_{11}$ $\quad R_{12} = 1.5(.1 B_{11}) = .15 B_{11}$ $\quad B_{12} = B_{11}(1.1) - .15 B_{11} = .95 B_{11}$

$\therefore B_{12} = .95(.95 B_{10}) = (.95)^2 B_{10}$

$\vdots \qquad \vdots \qquad \vdots$

Continue: $B_{20} = (.95)^{10} \cdot B_{10} = 1000(.95)^{10}$

For the last 10 payments, we have



$B_{20} = 1000(.95)^{10} = X\, a_{\overline{10}|.10}$

$\Longrightarrow X \doteq 97.44$

42/ May 2003 Course 2 Exam

$r = .08$

$i = .06$

$F = 10000$

$$I_7 = i \cdot B_6$$

$$= .06\left[800\, a_{\overline{4}|.06} + 10000\, v^4_{.06}\right] \doteq 641.58$$

2 / May 2005 Course FM Exam

$R^I = 10000(.09) = 900$

$R^{SF} \cdot s_{\overline{10}|.08} = 10000 \Rightarrow R^{SF} \doteq 690.29$

$R = R^I + R^{SF} = 1590.29$

Total paid by Lori over the 10-year period is

$X = 10R \doteq 15902.90$

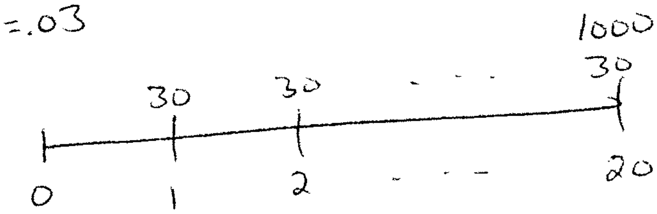
5/ (May 2005 Course FM Exam)

$i = \text{aeir}$

Zero-Coupon Bond: $624.60 = 1000\, v_i^{12} \Rightarrow i \doteq .04$ aeir

Coupon Bond: $r = .03$

$j = \text{seir} = (1.04)^{1/2} - 1$



semi-annual periods

$$X = 30\, a_{\overline{20}|j} + 1000\, v_j^{20} \doteq 1167.04$$

8/ May 2005 Course FM Exam

$B_{10} \overset{Pro}{=} 300\, a_{\overline{15}|.08} \doteq 2567.84$

After making the extra payment, the new balance is 1567.84.

$$\therefore\ 1567.84 = R\, a_{\overline{10}|.08} \Rightarrow R \doteq 233.65$$

11 / May 2005 Course FM Exam) Since the investor buys the bond at the highest price to guarantee her desired yield, then she bought at $P = 897$.

The bond is called after 20 years. We have

$$897 = 80\, a_{\overline{20|}i} + 1050\, v_i^{20}$$

$$\overset{TVM}{\Longrightarrow} i \doteq 9.24\%$$

25/ May 2005 Course FM Exam) $j_{eir} = .04$

$$B_3 = 500(1.04)^3 - 20\, s_{\overline{3}|.04} = 500$$

$$I_4 = i \cdot B_3 = .04(500) = 20$$

$$\therefore P_4 = 0$$

(This loan lasts <u>forever</u>; analagous to a perpetuity)

4 / Nov. 2005 Exam FM

$F = 100$

$r = .04$

$i = .03$

$n = 20$

$$118.20 = 4\, a_{\overline{20}|.03} + C\, v_{.03}^{20} \Rightarrow C \doteq 106$$

11/ Nov. 2005 Exam FM) Let X = amount borrowed

For the bond: $r = .04$ and $i = .03$

$$\therefore \quad X = 40\, a_{\overline{20}|.03} + 1000\, v_{.03}^{20} \doteq 1148.77$$

At the end of 10 years, the investor repays

$$X(1.05)^{10} \doteq 1871.23$$

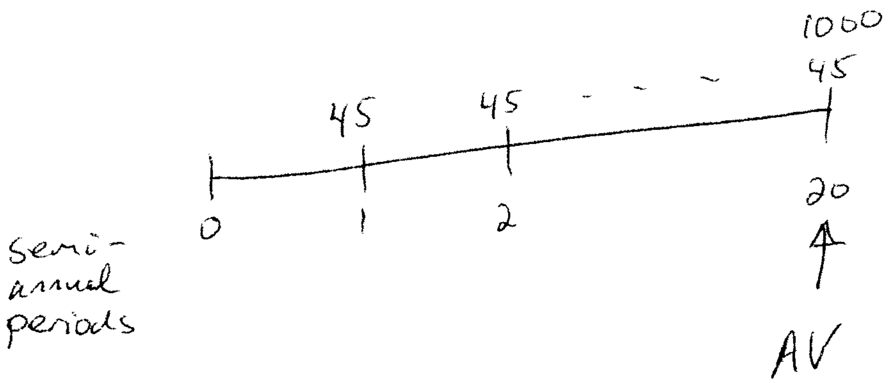
The investor has, at the end of 10 years,

$$40\, s_{\overline{20}|.02} + 1000 \doteq 1971.89$$

$$\therefore \text{ net gain} \doteq 1971.89 - 1871.23 \doteq 100.66$$

$r = .045$

16 / Nov. 2005 Exam FM) $P = 925$ = amount Dan paid at $t=0$



Invests coupons at .035 seir

$$AV = 45\, s_{\overline{20}|.035} + 1000 \doteq 2272.59$$

His yield over 10 year period as $i^{(2)}$ is determined by

$$AV = P\left(1 + \frac{i^{(2)}}{2}\right)^{20}$$

$$\therefore\ 2272.59 = 925\left(1 + \frac{i^{(2)}}{2}\right)^{20}$$

$$\Longrightarrow i^{(2)} \doteq 9.2\%$$

18 / Nov. 2005 Exam FM

$I_8 = 789 \qquad P_8 = 211$

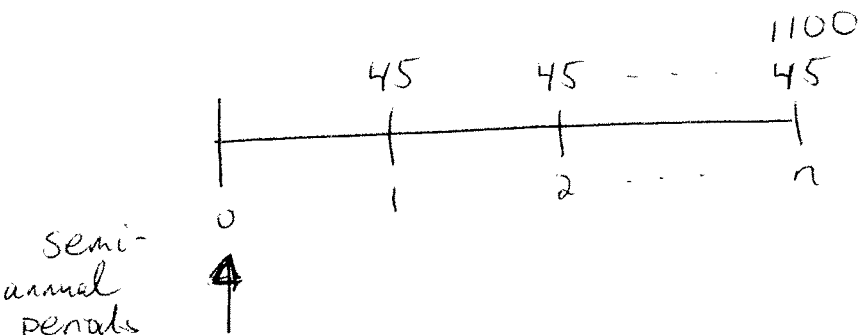
$\therefore R = 1000$ (level)

$$P_{18} = P_8 (1+i)^{10} = 211 (1.07)^{10}$$

$$\therefore \quad I_{18} = R - P_{18} \doteq 585$$

22 / Nov. 2005 Exam FM

$F = 1000$
$r = .045$ } $Fr = 45$
$i = .05$



$$P = 918 = 45\, a_{\overline{n}|.05} + 1100\, v^{n}_{.05} \xrightarrow{TVM} n \doteq 49.35$$

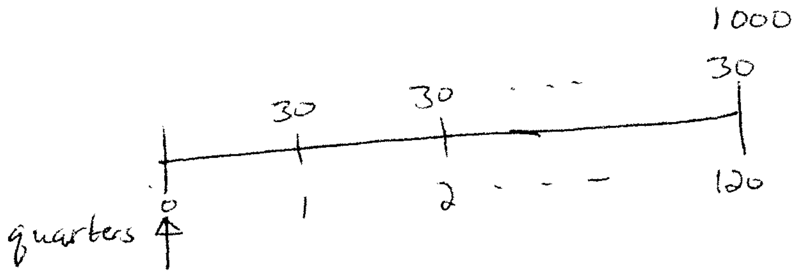
49.35 semiannual periods corresponds to

24.675 years

Best Answer Choice: (B) 25

24/ Nov. 2005 Exam FM

$r = .03$



$$P = 850 = 30\,a_{\overline{120}|i} + 1000\,v_i^{120}$$

$$i = \text{qeir} \doteq .0354$$

$$i^{(4)} = 4i \doteq 14.2\%$$

28 / Exam FM Sample Questions

$R = 1$

$$I_t = i \cdot B_{t-1} = i \cdot a_{\overline{n-(t-1)}|} = i \, \frac{1 - v^{n-t+1}}{i} = 1 - v^{n-t+1}$$

$$P_{t+1} = R - I_{t+1} = 1 - I_{t+1}$$

$$I_{t+1} = i \cdot B_t = i \cdot a_{\overline{n-t}|} = i \, \frac{1 - v^{n-t}}{i} = 1 - v^{n-t}$$

$$\therefore P_{t+1} = 1 - (1 - v^{n-t}) = v^{n-t}$$

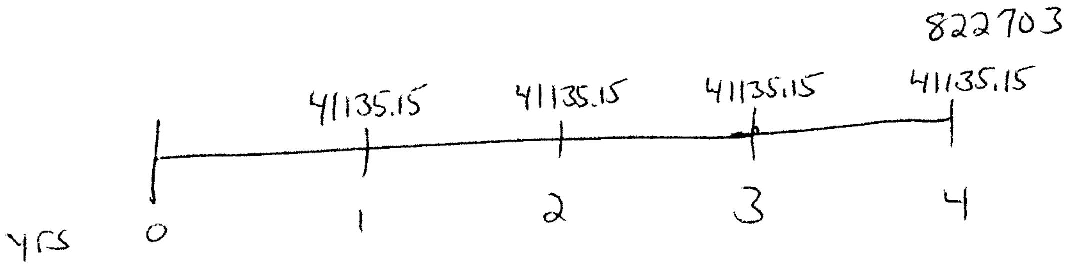
$$\therefore I_t + P_{t+1} = 1 - v^{n-t+1} + v^{n-t}$$

rewrite $1 + v^{n-t} - v^{n-t+1}$ $\qquad v^{n-t+1} = v^{n-t} \cdot v$

factor $1 + v^{n-t}(1 - v)$ $\qquad 1 - v = d$

$$= 1 + v^{n-t} \cdot d$$

30 / Exam FM Sample Questions) This is an easy question once we understand what the company is doing. Think of $r = .05$ and $F = 822703$, and so we get



The company anticipates reinvestment rates to be 5%. Then at time 4, they have

$$AV = 41135.15\, s_{\overline{4}|.05} + 822703 \doteq 1000000$$

exactly what they need!

The question asks, what happens if reinvestment rates are 4.5% or 5.5% instead. At 4.5%, they will have $AV = 41135.15\, s_{\overline{4}|.045} + 822703 \doteq 998{,}687$, but they need 1000000. They will have a loss of

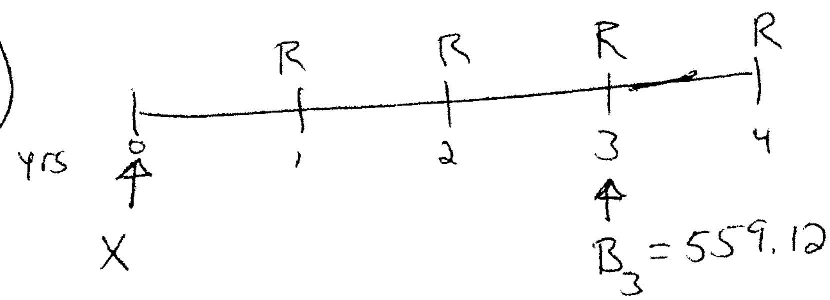
$$1000000 - 998687 = 1313$$

From here we get answer choice D.

You can show that at 5.5% reinvestment rates, the company will have a gain of 1323.

.08=aeir

46/ Exam FM Sample Questions



$$\therefore \quad 559.12 \underset{VEP}{\overset{Pro}{=}} R \cdot v_{.08} \implies R = 559.12(1.08)$$

$$\therefore X = R\, a_{\overline{4}|.08} \doteq 2000$$

$$I_1 = .08X \doteq 160$$

$$P_1 = R - I_1 \doteq 443.85$$

54/ Exam FM Sample Questions

$r = .04$

$i = \text{seir (yield)} \geq .03 \text{ seir}$

$$1722.25 = .04X\, a_{\overline{n}|i} + X\, v_i^n$$

Set up a table:

Call Time $= n$	X (to yield the guarantee of .03 seir)
30	$1722.25 = X(.04\, a_{\overline{30}|.03} + v_{.03}^{30}) \Rightarrow X = 1440$
31	$1722.25 = X(.04\, a_{\overline{31}|.03} + v_{.03}^{31}) \Rightarrow X = 1435$
32	Continue $X = 1430$
$\vdots$	
39	$X = 1402$
40	$X = 1399$

X's are decreasing (answer will be an extreme value)

Think: If $X = 1399$ but the bond is called at time $n = 30$, then we would have

$$1722.25 = .04(1399)\, a_{\overline{30}|i} + 1399\, v_i^{30} \Rightarrow i = 2.84,$$

but then we would not have the guaranteed yield of 3%. So X cannot equal 1399.

On the other hand, if $X = 1440$ but the bond is called at time $n = 40$, say, then we would have

$$1772.25 = .04(1440)\, a_{\overline{40}|i} + 1440\, v_i^{40} \Rightarrow i = 3.13\%,$$

above the guarantee of 3%, and so OK.

You may check that if $X = 1440$, then no matter when the bond is called, the yield is greater than or equal to 3%. Answer $X = 1440$.

55 / Exam FM Sample Questions

n	$P(.03)$
30	$40a_{\overline{30}\|.03} + 1200v_{.03}^{30} = 1278.40$
$\vdots$	$\vdots$
39	$40a_{\overline{39}\|.03} + 1200v_{.03}^{39} = 1291.23$
40	$40a_{\overline{40}\|.03} + 1100v_{.03}^{40} = 1261.80 \leftarrow$

The maximum price to guarantee a yield of at least 3% semiannual effective is 1262.

56 / (Exam FM Sample Questions)

Coupon amount $= .02X$

$i = .03 = \text{seir}$

n	$P(.03) = 1021.50)$
10	$1021.50 = .02X\,a_{\overline{10}\vert .03} + X v^{10}_{.03} = X(.02\,a_{\overline{10}\vert .03} + v^{10}_{.03}) \Rightarrow X = 1116.76$
$\vdots$	$\vdots$
20	$1021.50 = .02X\,a_{\overline{20}\vert .03} + X v^{20}_{.03} \Longrightarrow X = 1200.03$

Think! If $X = 1116.76$ and the bond is held to maturity $(n=20)$ then what is the yield? $1021.50 = .02(1116.76)a_{\overline{20}\vert i} + 1116.76\,v_i^{20}$

$\overset{\text{TVM}}{\Longrightarrow} i = 2.55\% < 3\%$, which is supposed to be Sue's lowest yield. $\therefore X \neq 1116.76$

Answer: $X = 1200.03$ (or just 1200)

<u>Note</u>: If the bond is called at time $n=10$, then Sue's yield is determined as

$$1021.50 = .02(1200)\,a_{\overline{10}\vert i} + 1200\,v_i^{10} \overset{\text{TVM}}{\Longrightarrow} i \doteq 3.8\%,$$

which is consistent with Sue getting a yield of at least $i = 3\%$.

57 / Exam FM Sample Questions

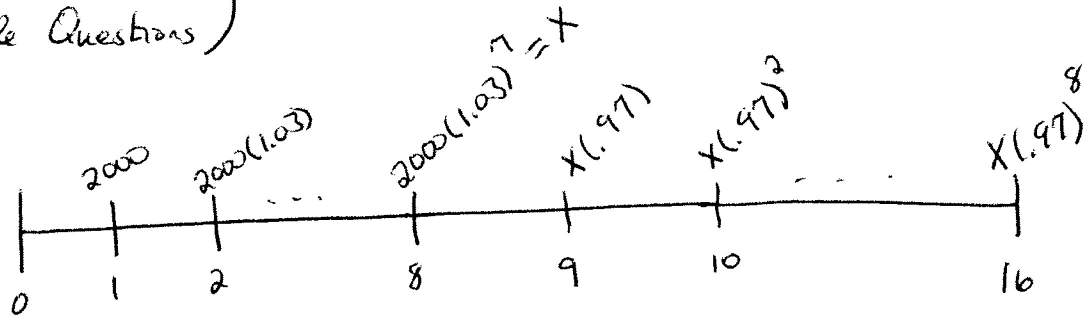
coupon amount $= 1100(.02) = 22$

n	$P(i) = 1021.50$
10	$1021.50 = 22a_{\overline{10}|i} + 1200v_i^{10} \xrightarrow{TVM} i \doteq 3.63\%$
⋮	⋮
19	$1021.50 = 22a_{\overline{19}|i} + 1200v_i^{19} \xrightarrow{TVM} i \doteq 2.86\%$
20	$1021.50 = 22a_{\overline{20}|i} + 1100v_i^{20} \xrightarrow{TVM} i \doteq 2.46\%$

$\therefore$ Mary's minimum yield is $i = 2.46\%$ seir

$\Longrightarrow i^{(2)} = 2i = 4.92\%$ (or 4.9%)

60/ Exam FM Sample Questions



aeir = .07

$$L \overset{VEP}{=} 2000v + 2000(1.03)v^2 + \cdots \text{ (8 payments/terms)}$$

$$+ X(.97)v^9 + X(.97)^2 v^{10} + \cdots \text{ (8 payments/terms)}$$

$$= \frac{2000}{1.07}\left(1 + \frac{1.03}{1.07} + \cdots \text{ (8 terms)}\right)$$

$$+ \underbrace{2000(1.03)^7}_{=X}(.97)(1.07)^{-9}\left(1 + \frac{.97}{1.07} + \cdots \text{ (8 terms)}\right)$$

$$\therefore L = \frac{2000}{1.07}\cdot \ddot{a}_{\overline{8}|\left(\frac{1.07}{1.03}-1\right)} + 2000(1.03)^7(.97)(1.07)^{-9}\cdot \ddot{a}_{\overline{8}|\left(\frac{1.07}{.97}-1\right)}$$

$$= 20688.63$$

62/(Exam FM Sample Questions)

$$\sum_{k=1}^{40} P_k = P_1 + P_2 + \cdots + P_{40} = P - C$$

$P_{15} = -194.82 \qquad P_{20} = -306.69$

$P_{20} = P_{15}(1+i)^5 \Longrightarrow i \doteq .095$

$P_1 = P_{15} \cdot v_i^{14}$

$$\sum_{k=1}^{40} P_k = P_1 + P_2 + \cdots + P_{40}$$
$$= P_1\left(1 + (1+i) + \cdots + (1+i)^{39}\right) \overset{VEP}{=} P_1 \cdot s_{\overline{40}|i}$$

$$\therefore \; P - C = P_1 \cdot s_{\overline{40}|i} = \underbrace{-194.82 \cdot v_i^{14} \cdot s_{\overline{40}|i}}_{\text{this is negative because the bond was bought at a discount}}$$

The <u>amount</u> of discount is

$$194.82\, v_i^{14} \cdot s_{\overline{40}|i} \doteq 21{,}135$$

63 / (Exam FM Sample Questions)

$\sum_{K=1}^{8} P_K = L$. Total amount of interest paid is $8R - L$.

$P_5 = 699.68$ $\quad i = .0475$

$P_1 = P_5 \cdot v_i^4$

$$\sum_{K=1}^{8} P_K = P_1 + P_2 + \cdots + P_8$$
$$= P_1 (1 + (1+i) + \cdots + (1+i)^7) \overset{\text{VEP}}{=\!=} P_1 \cdot s_{\overline{8}|i}$$

$$\therefore L = P_1 \cdot s_{\overline{8}|i} = 699.68 \cdot v^4 \cdot s_{\overline{8}|} \doteq 5500$$

Also, $L = R\, a_{\overline{8}|i} \implies R \overset{\text{TVM}}{=\!=} 842.39$

$\therefore$ the total amount of interest paid is

$8R - L \doteq 1239$

64 / Exam FM Sample Questions) (We don't need to find n.)

$$B_{18} = 16{,}337.10$$

Keystrokes END 18 $\boxed{N}$ 8.4 $\boxed{\div}$ 12 $\boxed{=}$ $\boxed{I/Y}$ 22000 $\boxed{PV}$ 450.3 $\boxed{+/-}$ $\boxed{PMT}$ $\boxed{CPT}$ $\boxed{FV}$

We determine the new payment using

$$16337.10 = R\, a_{\overline{24}|m} \qquad m = \text{meir} = \frac{.048}{12}$$

$$\Longrightarrow R \doteq 715.27$$

65/Exam FM Sample Questions

$F = C = 5000$

$r = \frac{.076}{2} = .038$

$n = 14$

$P = 5000$ (since no premium or discount)

↳ this is also referred to as buying at par, which means $P = C$

Coupons are $Fr = 190$

From the basic pricing formula,

$$5000 = 190\, a_{\overline{14|}i} + 5000\, v_i^{14} \implies i = .038 \; (= r)$$

Since $F = C$ and $r = i$, we can use the shortcut for the MacD of the bond; namely,

$$\text{MacD} = \ddot{a}_{\overline{14|}i} \doteq 11.11$$ (the time unit here is in semi-annual periods, since the coupons are paid semi-annually.)

☆ In years, $\text{MacD} \doteq \frac{11.11}{2} \doteq 5.56$

Remark: If you don't recognize that you can use the shortcut, then

$$\text{MacD} = \frac{190 (Ia)_{\overline{14|}} + 5000(14) v^{14}}{190\, a_{\overline{14|}} + 5000\, v^{14}} \overset{i=.038}{=\!=} 11.11 \text{ as above}$$

74 / (Exam FM Sample Questions)

Bond A: $r = \frac{i}{2} + .02 \Rightarrow$ coupons $= 5000i + 200$

Bond B: $r = \frac{i}{2} - .02 \Rightarrow$ coupons $= 5000i - 200$

$$P^A = (5000i + 200)\, a_{\overline{20}|\, i/2} + 10000\, v_{i/2}^{20}$$

$$-\; P^B = (5000i - 200)\, a_{\overline{20}|\, i/2} + 10000\, v_{i/2}^{20}$$

$$\therefore\; P^A - P^B = 400\, a_{\overline{20}|\, i/2} = 5341.12$$

$$\xRightarrow{\text{TVM}} i \doteq .084$$

75 / (Exam FM Sample Questions)

$$\frac{.09}{12} = .0075$$

$$400000 = R\, a_{\overline{180}|.0075} \implies R = 4057.07 \ \left(\begin{array}{c}\text{original}\\ \text{payments}\end{array}\right)$$

$$B_{36} = R\, a_{\underbrace{\overline{180-36}}_{=144}|.0075} \quad \left(\begin{array}{c}\text{this is prospective;}\\ \text{could have used retrospective}\end{array}\right)$$

$$\implies B_{36} \doteq 356{,}498.85$$

$$R^{new} = 4057.07 - 409.88 = 3647.19$$

$$\therefore\ 356498.85 = 3647.19\, a_{\overline{144}|\, j/12}$$

$$\implies \frac{j}{12} \doteq .575\,\%$$

$$\implies j \doteq 6.9\%$$

76 / (Exam FM Sample Questions)

For the first bond,

$$P = 25\, a_{\overline{60}|,025} + 1200\, v^{60}_{,025} = 1045.46$$

$\therefore$ for the second bond

$$1045.46 = 25\, a_{\overline{60}|\, j/2} + 800\, v^{60}_{j/2}$$

$$\xRightarrow{\text{TVM}} \frac{j}{2} \doteq 2.2\% \implies j \doteq 4.4\%$$

80 / Exam FM Sample Questions

$$R^{SF} = 2(1000i) = 2000i$$

$$\therefore \quad 2000i \cdot s_{\overline{5}|0.8i} = 1000$$

$$\Rightarrow 2000\cancel{i}\,\frac{(1+0.8i)^5-1}{0.8\cancel{i}} = 1000$$

$$\Rightarrow (1+0.8i)^5 = 1.4 \Rightarrow i \doteq 8.7\%$$

81 / (Exam FM Sample Questions) $\quad I_1 = i \cdot L$

$$P_{26} = X = B_{25} - B_{26}$$

Equation #

(1) $\quad L = 2500\, a_{\overline{26}|} + B_{26} \cdot v^{26}$

(2) $\quad L = 2500\, a_{\overline{25}|} + B_{25} \cdot v^{25}$

If we subtract the second equation from the first, we would have as one of the terms, $B_{26} \cdot v^{26} - B_{25} \cdot v^{25}$, which doesn't ~~factor~~. So, let's first multiply the first equation by $(1+i)$ and then subtract. We get

$$\begin{array}{rl} (1+i) \cdot L = & 2500 \cdot (1+i) a_{\overline{26}|} + B_{26} \cdot v^{25} \\ - \quad L = & 2500\, a_{\overline{25}|} + B_{25} \cdot v^{25} \\ \hline \end{array}$$

$$\star \quad \underbrace{(1+i) \cdot L - L}_{= i \cdot L, \text{ which is what we seek! } (i \cdot L = I_1)} = 2500\left((1+i)\, a_{\overline{26}|} - a_{\overline{25}|}\right) + \underbrace{(B_{26} - B_{25})}_{= -X} \cdot v^{25}$$

Now let's focus on $(1+i)\, a_{\overline{26}|}$. Using the VEP expression to simplify, we get

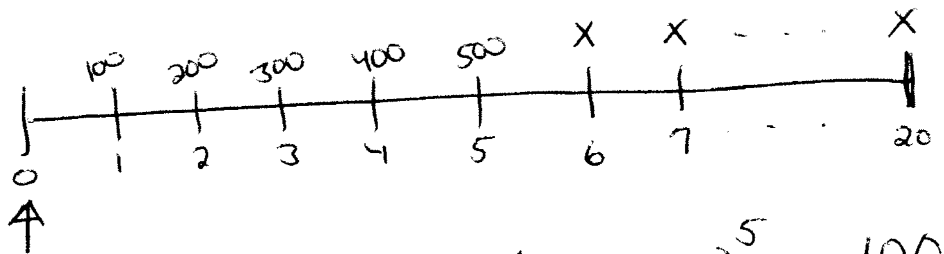
$$(1+i)\, a_{\overline{26}|} = (1+i) \cdot (v + v^2 + v^3 + \cdots + v^{26}) = 1 + \underbrace{v + v^2 + \cdots + v^{25}}_{= a_{\overline{25}|}}$$

$$\therefore (1+i)\, a_{\overline{26}|} = 1 + a_{\overline{25}|} \implies (1+i)\, a_{\overline{26}|} - a_{\overline{25}|} = 1$$

$$\therefore \text{ from } \star, \quad i \cdot L = I_1 = 2500(1) - X v^{25}$$

86 / (Exam FM Sample Questions)

aeir = .05



$$L = 100(Ia)_{\overline{5}|.05} + X\, a_{\overline{15}|.05} \cdot v_{.05}^{5} = 10000$$

$$\Longrightarrow X = 1075.08$$

88/ Exam FM Sample Questions

$$65000 = R\,a_{\overline{180}|\frac{.08}{12}} \Rightarrow R \doteq 621.17 \left(\begin{matrix}\text{original}\\ \text{payments}\end{matrix}\right)$$

$$B_{12} = R\,a_{\underbrace{\overline{180-12}|}_{=168}\frac{.08}{12}} \doteq 62661.40$$

$$\therefore 62661.40 = R^{new}\,a_{\overline{168}|\frac{.06}{12}} \Rightarrow R^{new} \doteq 552.19$$

90 / Exam FM Sample Questions

$F = 1000$

$r = \frac{.09}{2} = .045$ $\Big\rangle$ $Fr = 45$

$n = 40$

$C = 1200$

$i = \frac{.1}{2} = .05$

$$P = 45\, a_{\overline{40}|.05} + 1200\, v_{.05}^{40} = 942.61$$

91 / (Exam FM Sample Questions)

$F = 1000 = C$

$r = \frac{.10}{2} = .05$

$i = \frac{.12}{2} = .06$

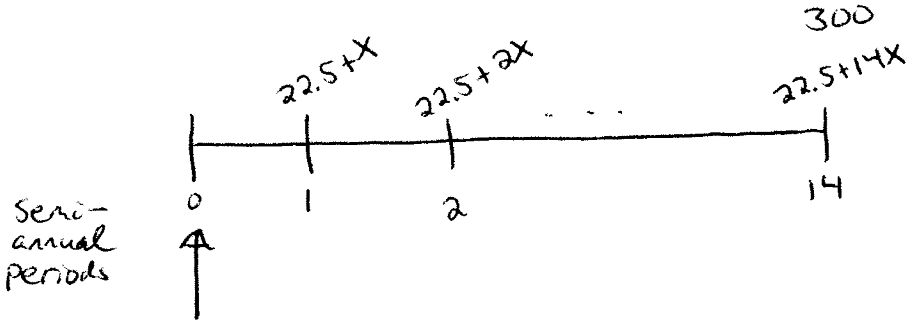
$Fr = 50$

n	$P(.06)$
2	$50\,a_{\overline{2}\rceil .06} + 1000\,v^{2}_{.06} = 981.67$
$\vdots$	$\vdots$
20	$50\,a_{\overline{20}\rceil .06} + 1000\,v^{20}_{.06} = 885.30$

Any price above 885.30 is paying too much to yield $i = .06$ seir

100/Exam FM Sample Questions

seir = .03

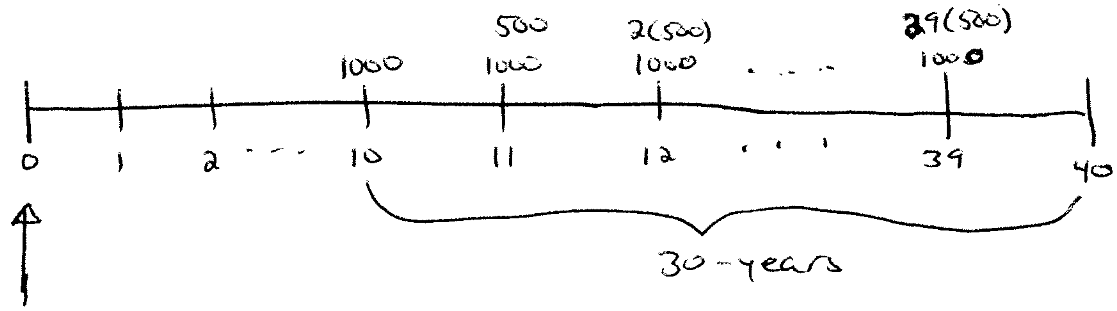


$$PV = 1050.50 = 22.5\, a_{\overline{14|}.03} + X \cdot (Ia)_{\overline{14|}.03} + 300\, v^{14}_{.03}$$

$$\Longrightarrow X = 7.54$$

101 / Exam FM Sample Questions

aeir = .05

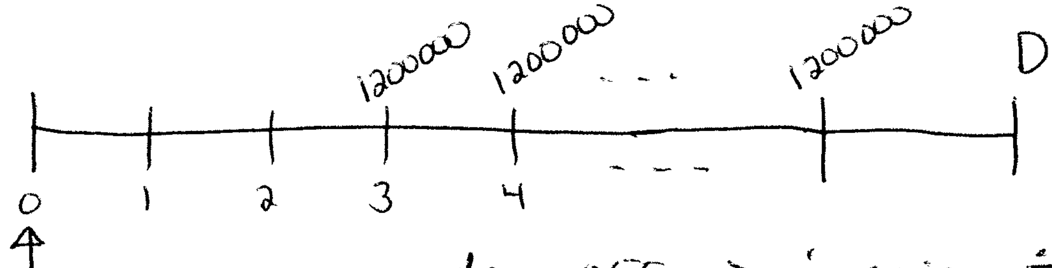


$$L = 1000\, a_{\overline{30|}} \cdot v^{9} + 500\,(Ia)_{\overline{29|}} \cdot v^{10}$$

$$= \cancel{6420}\ 64257.02$$

106 / (Exam FM Sample Questions)

D = amount of "drop payment" $\underbrace{\text{drop payment}}_{\text{extra payment that pays off the loan}}$



$aedr = .055 \Rightarrow i = aeir = \frac{.055}{1-.055}$

L = 15,000,000

Changing the valuation date to $t=2$ allows us to find the number of payments of 1200000 as follows

$$15000000(1-.055)^{-2} = 1200000\, a_{\overline{n}|i}$$

$$\Rightarrow n = 29+$$

$$\therefore\ 15000000(1-.055)^{-2} = 1200000\, a_{\overline{29}|i} + D \cdot v^{30}$$

$$\Rightarrow D = 960723.59$$

Remark: The SOA solution uses rounded number; e.g. the solution uses $i = 5.82\%$ whereas this solution does not use this rounded value. This is why the value of D is a little off from the SOA solution.

107/ Exam FM Sample Questions

$$I_{n-1} = .525\, I_{n-3} = .1427\, I_1$$

Since $I_K = i \cdot B_{K-1}$, we can write the above equations in terms of balances as follows:

$$B_{n-2} = .525\, B_{n-4} = .1427\, B_0$$

Note: 1) $B_{n-4} = R\, a_{\overline{2|}} + B_{n-2} \cdot v^2$

2) $B_{n-2} \stackrel{\text{Pro}}{=} R\, a_{\overline{2|}}$

$$\therefore B_{n-4} = B_{n-2} + B_{n-2} \cdot v^2 = B_{n-2}(1+v^2)$$

We're given $B_{n-4} = \frac{1}{.525} \cdot B_{n-2}$

$$\therefore 1+v^2 = \frac{1}{.525} \Rightarrow i \doteq 5.1315\%$$

We also have $B_{n-2} = .1427\, B_0$

$$\Rightarrow \cancel{R}\, a_{\overline{2|}} = .1427(\cancel{R}\, a_{\overline{n|}}) \Rightarrow a_{\overline{n|}} = \frac{a_{\overline{2|}}}{.1427}$$

$$\Rightarrow \frac{1-v^n}{i} = \frac{a_{\overline{2|}}}{.1427} \Rightarrow v^n = 1 - i\,\frac{a_{\overline{2|}}}{.1427}$$

Since $i \doteq 5.1315\%$, then $n \doteq 22$

108 / Exam FM Sample Questions

i) $L = R^{SF} \cdot s_{\overline{11}|.047}$

ii) $L - AV_7^{SF} = 6241$

$AV_7^{SF} = R^{SF} \cdot s_{\overline{7}|.047}$

$\therefore \; 6241 = R^{SF} \cdot s_{\overline{11}|} - R^{SF} s_{\overline{7}|} = R^{SF}\left(s_{\overline{11}|} - s_{\overline{7}|} \right)$

$$\Rightarrow R^{SF} = \frac{6241}{s_{\overline{11}|.047} - s_{\overline{7}|.047}} = 1054.57$$

109 / Exam FM Sample Questions

$meir = \frac{.06}{12} = .005 = m$

$200000 = R a_{\overline{360}|m} \Rightarrow R = 1199.10$ (amount of monthly payments)

As of 12/31/2007 we have:

1) If there were no extra payments made, then

$$B_{60}^{(\text{no extra payments})} = R a_{\overline{300}|m} = 186108.72$$

2) The accumulated value of the extra payments is

$$AV^{(\text{extra payments})} = 10000\, s_{\overline{5}|i} \quad i = aeir = (1.005)^{12} - 1$$
$$= 56{,}560.07$$

∴ as of 12/31/2007, with the extra payments,

$$B_{12/31/2007} = 186108.72 - 56560.07 = 129548.65$$

We get the number of remaining payments as follows:

$$129548.65 = 1199.10\, a_{\overline{n}|.005} \Rightarrow n = 155+$$

The drop payment is made 156 months (exactly 13 years) after 12/31/2007, which is 12/31/2020

110/ (Exam FM Sample Questions)

aedr = .08

$$\Rightarrow \text{aecr} = \frac{.08}{1-.08} = \frac{8}{92} = i$$

$$500000 = R\, a_{\overline{5}|i} \Rightarrow R = 127532.72$$

We calculate X by solving

$$500000 = 128000\, a_{\overline{4}|i} + X \cdot v_i^5$$

$$\Rightarrow X = 125{,}220.38$$

112/Exam FM Sample Questions

i) $I_1 = 3600 = i \cdot B_0 = i \cdot (R a_{\overline{10|}}) = R(1-v^{10})$

since $a_{\overline{10|}} = \frac{1-v^{10}}{i}$

$\therefore\ 3600 = R(1-v^{10})$

ii) $P_6 = 4871$

$P_6 = R - I_6 = R - i \cdot B_5 = R - [\underbrace{i(R a_{\overline{5|}})}_{=R(1-v^5)}]$

$\therefore\ P_6 = R - [R(1-v^5)] = \cancel{R} - [\cancel{R} - Rv^5]$

$\Rightarrow P_6 = Rv^5 \Rightarrow 4871 = Rv^5$

$\therefore \begin{cases} 3600 = R(1-v^{10}) \\ 4871 = Rv^5 \Rightarrow R = \frac{4871}{v^5} \end{cases}$

$\therefore\ 3600 = \frac{4871}{v^5}(1-v^{10}) \Rightarrow 3600v^5 = 4871 - 4871v^{10}$

$\Rightarrow 4871v^{10} + 3600v^5 - 4871 = 0 \xrightarrow[\text{in } v^5]{\text{quadratic}} v^5 \doteq .69656$

$\Rightarrow i \doteq .075$ and $R = \frac{4871}{v^5} \doteq 6992.94$

$X = R a_{\overline{10|}i} \doteq 48000$

113 / Exam FM Sample Questions

$C = F$

$r = \frac{.07}{2} = .035$

$aeir = .0705 \Rightarrow i = seir = (1.0705)^{1/2} - 1$

$n = 50$

$$10000 = F(.035)\, a_{\overline{50}|i} + F\, v_i^{50}$$

$$\Rightarrow C = F = \frac{10000}{.035\, a_{\overline{50}|i} + v_i^{50}} = 9917.99$$

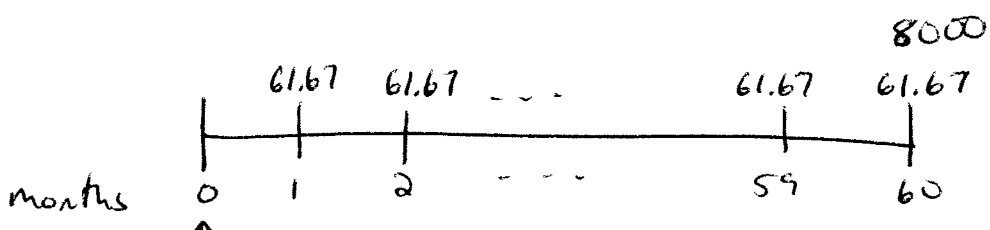
114/Exam FM Sample Questions

At time 0, Jeff invests 8000 of his own money.

For the next 10 years, at the end of each month, he receives a coupon of $10000(.0075)=75$, but makes an interest payment of $2000\left(\frac{.08}{12}\right)=13.33$. So he receives $75-13.33=61.67$ each month for 10 years.

At the end of 10 years, the bond matures for 10000 but the loan amount of 2000 is due. So he receives 8000.

$\therefore$ The timeline is



$$PV = 8000 = 61.67\, a_{\overline{60}|m} + 8000\, v_m^{60}$$

$\therefore$ Jeff's monthly effective yield is $m \overset{TVM}{=} 0.00770875$

$\Rightarrow$ his annual effective yield rate is $i=(1+m)^{12}-1 \doteq 9.65\%$

115 / Exam FM Sample Questions

$$\left.\begin{aligned} P^{I} &= 52.8\, a_{\overline{n}|} + 1000\, v^n \\ -\ P^{II} &= 48.4\, a_{\overline{n}|} + 1100\, v^n \end{aligned}\right\} P^{I} = P^{II}$$

$$0 = 4.4\, a_{\overline{n}|} - 100\, v^n \implies v^n = .044\, a_{\overline{n}|}$$

$$\left.\begin{aligned} P^{III} &= 1320r\, a_{\overline{n}|} + 1320\, v^n \\ -\ P^{I} &= 52.8\, a_{\overline{n}|} + 1000\, v^n \end{aligned}\right\} P^{III} = P^{I}$$

$$0 = a_{\overline{n}|}(1320r - 52.8) + 320\, v^n$$

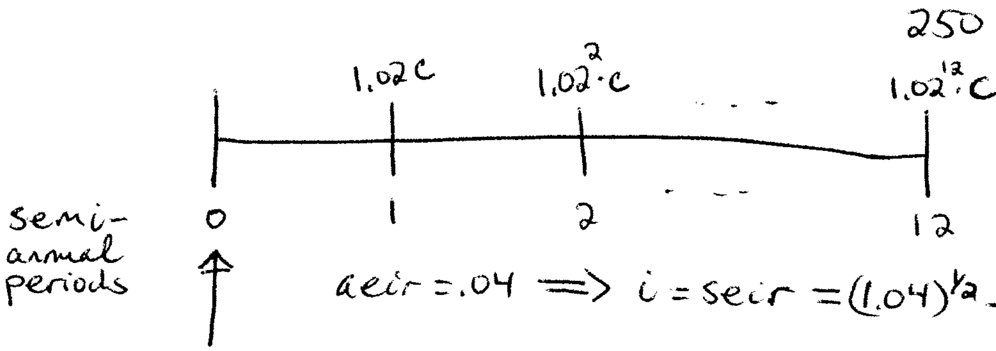
$$v^n = .044\, a_{\overline{n}|} \implies 320\, v^n = 14.08\, a_{\overline{n}|}$$

$$\therefore 0 = a_{\overline{n}|}(1320r - 52.8 + 14.08)$$

$$\implies 1320r - 52.8 + 14.08 = 0$$

$$\implies r = .029\overline{3}$$

116/Exam FM Sample Questions



$aeir = .04 \implies i = seir = (1.04)^{1/2} - 1$

$$PV = 582.53 \overset{\text{VEP}}{=} 1.02c \cdot v + 1.02^2 \cdot c \cdot v^2 + \cdots + 1.02^{12} \cdot c \cdot v^{12} + 250v^{12}$$

$$= \frac{1.02c}{1+i}\Big(1 + \underbrace{\frac{1.02}{1+i}}_{r>1} + \cdots (12 \text{ terms})\Big) + 250v^{12}$$

$$\therefore\ 582.53 = \frac{1.02c}{1+i} \cdot s_{\overline{12}|\left(\frac{1.02}{1+i}-1\right)} + 250v_i^{12}$$

$$i = (1.04)^{1/2} - 1$$

$$\implies c \doteq 32.04$$

117 / Exam FM Sample Questions

$C = 1890$
$i = .06$

$$B_3 = Fr\, a_{\overline{1|}} + B_4 \cdot v$$

$$\Rightarrow 1254.87 = Fr\, a_{\overline{1|}} + 1277.38\, v_{.06} \Rightarrow Fr = 52.7822$$

$$P = Fr\, a_{\overline{4|}} + B_4 \cdot v^4 \Rightarrow P \doteq 1194.70$$

$$P = Fr\, a_{\overline{n|}} + C\, v^n \Rightarrow n = 20$$

118 / Exam FM Sample Questions

$F = C = 2500$ $\qquad$ $Fr = 87.5$

$r = \frac{.07}{2} = .035$

$i = \frac{.08}{2} = .04$

$$B_4 = B_3(1+i) - Fr = B_3 + .04B_3 - 87.5$$

$$\Rightarrow B_4 - B_3 = .04B_3 - 87.5 \overset{\text{by}}{\underset{\text{iv)}}{=\!=}} 8.44$$

$$\Rightarrow B_3 = 2398.50$$

$$\therefore \quad P = Fr\, a_{\overline{3}|} + B_3 \cdot v^3 = 87.5\, a_{\overline{3}|,.04} + 2398.50\, v^3_{.04}$$

$$\Rightarrow P = 2375.08$$

Then $\quad P = Fr \cdot a_{\overline{\#\text{of coupons}}|} + C \cdot v^{\#\text{ of coupons}}$

We normally use n to denote the number of coupons, but since we seek n, and it denotes the number of years, we have # of coupons $= 2n$

$$\therefore \; 2375.08 = 87.5\, a_{\overline{2n}|,.04} + 2500\, v^{2n}_{.04} \Rightarrow 2n \overset{\text{TVM}}{=\!=} 13$$

$$\Rightarrow n = 6.5$$

Remark: We didn't really need to find P. We could have used B_3 as follows:

$$B_3 = Fr \cdot a_{\overline{2n-3}|} + C \cdot v^{2n-3} \Rightarrow 2n-3 \overset{\text{TVM}}{=\!=} 10$$

$$\Rightarrow n \doteq 6.5$$